Recurrent Neural Networks (RNN)

KING LEAR:
O, if you were a feeble sight, the courtesy of your law,
Your sight and several breath, will wear the gods
With his heads, and my hands are wonder'd at the deeds,
So drop upon your lordship's head, and your opinion
Shall be against your honour.

http://karpathy.github.io/2015/05/21/rnn-effectiveness/

Some slides from Richard Socher, Geoffrey Hinton, Andrej Karpathy
Motivating Example: Language Models

• Want to assign probability to a sentence
  • “Dafjdkf adkjfhalj fadlag dfah” – zero probability
  • “Furiously sleep ideas green colorless” – very low probability
  • “Colorless green ideas sleep furiously” – slightly higher
  • “The quick brown fox jumped over the lazy dog” – even higher

https://en.wikipedia.org/wiki/Colorless_green_ideas_sleep_furiously
Application for Language Models

• Applications
  • OCR gives several hypotheses, need to choose the most probable one
  • Choose a plausible translation from English to French
  • Complete the sentence “A core objective of a learner is to generalize from its […]”

• In every case, a language model can be used to evaluate all the possible hypotheses, and select the one with the highest probability
Sentence Completion

• Suppose a language model M can compute $P_M(w_1, w_2, ..., w_k)$

• For an incomplete sentence $w_1 w_2 w_3 ... w_{k-1}$, find $\text{argmax}_{w_k} P(w_1, w_2, w_3, ..., w_k)$ to complete the sentence

• Now, fix $w_k$, and find $\text{argmax}_{w_{k+1}} P(w_1, w_2, w_3, ..., w_k, w_{k+1})$
Probabilistic Sentence Generation

- \( P(w_k|w_1, w_2, ... w_{k-1}) = \frac{P(w_1 ... w_{k-1}w_k)}{P(w_1 ... w_{k-1})} \propto P(w_1 ... w_{k-1}w_k) \)

- Choose word \( w^{(j)} \) according to
  \[
  \frac{\exp(\alpha \hat{P}(w_1w_2 ... w_{k-1}w^{(j)}))}{\sum_j \exp(\alpha \hat{P}(w_1w_2 ... w_{k-1}w^{(j)}))}
  \]

- (Question: Higher \( \alpha \) => ?)

- Generally, take \( \hat{P} \) to be the input to the softmax that produces the probability in the RNN (better), or simply the probability of the sentence
Generating “Shakespeare” character-by-character with RNN

KING LEAR:
O, if you were a feeble sight, the courtesy of your law,
Your sight and several breath, will wear the gods
With his heads, and my hands are wonder'd at the deeds,
So drop upon your lordship's head, and your opinion
Shall be against your honour.

http://karpathy.github.io/2015/05/21/rnn-effectiveness/

(Here, the w’s are characters, not words)
Recurrent Neural Networks

\[ x_t - \text{the } t^{\text{th}} \text{ character of the string ("the character at time } t\text{")} \]

\[ h_t - \text{the } t^{\text{th}} \text{ character of the string ("the character at time } t\text{")} \]
RNN for Language Modelling

• Given a list of word vectors (e.g., one-hot encodings of words) \( x_1, \ldots, x_{t-1}, x_t, x_{t+1}, \ldots, x_T \)

At a single time step:

- \( h_t = \sigma(W^{(hh)} h_{t-1} + W^{(hx)} x_t) \)
- \( \hat{y}_t = \text{softmax}(W^{(S)} h_t) \)
- \( \hat{P}(x_{t+1} = v_j | x_1, x_2, \ldots, x_t) = \hat{y}_{t,j} \)

• \( h \) is the state (e.g., the previous word vector could be part of \( h \))
• \( x_t \) is the data
• \( \hat{y}_t \) is the predicted output
\(x_t\) — the input (one-hot) at \(t\)
\(\hat{y}_t\) — predictions (vector of probs) at \(t\)

\[
h_t = \sigma \left( W^{(hh)} h_{t-1} + W^{(hx)} x_t \right)
\]
\[
\hat{y}_t = \text{softmax} \left( W^{(s)} h_t \right)
\]
\[
\hat{p}(x_{t+1} = v_j | x_1, x_2, ..., x_t) = \hat{y}_{t,j}
\]
Cost Function

• Same as before: negative log-probability of the right answer:

\[ J^{(t)} = - \sum_{j=1}^{V} y_{t,j} \log \hat{y}_{t,j} \]

\[ J = \sum_{t} J^{(t)} \]

• \( \hat{y}_{t,j} = 1 \) iff \( x_{t+1} = v_j \)
Visualizing the hidden state*

Cell sensitive to position in line:

The sole importance of the crossing of the Berezina lies in the fact that it plainly and indubitably proved the fallacy of all the plans for cutting off the enemy's retreat and the soundness of the only possible line of action—the one Kutuzov and the general mass of the army demanded—namely, simply to follow the enemy up. The French crowd fled at a continually increasing speed and all its energy was directed to reaching its goal. It fled like a wounded animal and it was impossible to block its path. This was shown not so much by the arrangements it made for crossing as by what took place at the bridges. When the bridges broke down, unarmed soldiers, people from Moscow and women with children who were with the French transport, all—carried on by vis inertiae—pressed forward into boats and into the ice-covered water and did not surrender.

Karpathy et al. “Visualizing and Understanding Recurrent Networks”
http://arxiv.org/abs/1506.02078

*The RNN there is somewhat more complicated than what we saw so far
"You mean to imply that I have nothing to eat out of.... On the contrary, I can supply you with everything even if you want to give dinner parties," warmly replied Chichagov, who tried by every word he spoke to prove his own rectitude and therefore imagined Kutuzov to be animated by the same desire.

Kutuzov, shrugging his shoulders, replied with his subtle penetrating smile: "I meant merely to say what I said."

Karpathy et al. “Visualizing and Understanding Recurrent Networks”
http://arxiv.org/abs/1506.02078
A large portion of cells are not easily interpretable. Here is a typical example:

```c
/* Unpack a filter field's string representation from user-space */
char *audit_unpack_string(void **bufp, size_t *remain, size_t len)
{
    char *str;
    if (!*bufp || (len == 0) || (len > *remain))
        return ERR_PTR(-EINVAL);
    /* Of the currently implemented string fields, PATH_MAX defines the longest valid length. */
}
```

Karpathy et al. “Visualizing and Understanding Recurrent Networks”  
http://arxiv.org/abs/1506.02078
What can we do besides predicting the next letter in an English text?

• Predict the price of a stock tomorrow
• Predict the diagnosis of a patient tomorrow
• ...